

# Orbit determination of Transneptunian objects and Centaurs for the prediction of stellar occultations

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## ABSTRACT

**Context.** The prediction of stellar occultations by Transneptunian objects (TNOs) and Centaurs is a difficult challenge that requires accuracy both in the occulted star position as for the object ephemeris. Until now, the most used method of prediction involving tens of TNOs/Centaurs was to consider a constant offset for the right ascension and for the declination with respect to a reference ephemeris, usually, the latest public version. This offset is determined as the difference between the most recent observations of the TNO and the reference ephemeris. This method can be successfully applied when the offset remains constant with time, i.e. when the orbit is stable enough. In this case, the prediction holds even for occultations to occur several days after the last observations.

**Aims.** This paper presents an alternative method of prediction based on a new accurate orbit determination procedure, which uses all the available positions of the TNO from the Minor Planet Center database plus sets of new astrometric positions from unpublished observations.

**Methods.** The orbit determination is performed through a numerical integration procedure called NIMA, in which we develop a specific weighting scheme that takes into account the individual precision of observation, the number of observations performed during one night in a same observatory and the presence of systematic errors in the positions.

**Results.** The NIMA method was applied for 51 selected TNOs and Centaurs. For this purpose, we have performed about 2900 new observations in several observatories (European South Observatory, Observatório Pico dos Dias, Pic du Midi, etc) during the 2007-2014 period. Using NIMA, we succeed in predicting the stellar occultations of 10 TNOs and 3 Centaurs between July 2013 and February 2015. By comparing the NIMA and JPL ephemerides, we highlighted the variation of the offset between them with time, showing that in general the constant offset hypothesis is not valid, even for short time scales of a few weeks. Giving examples, we show that the constant offset method could not accurately predict 6 out of the 13 observed positive occultations successfully predicted by NIMA. The results indicate that NIMA is capable of efficiently refine the orbits of these bodies. Finally, we show that the astrometric positions given by positive occultations can help to further refine the orbit of the TNO and consequently the future predictions. We also provide the unpublished observations of the 51 selected TNOs and their ephemeris in a usable format by the SPICE library.

**Key words.** — Astrometry – Celestial mechanics – Occultations – Kuiper belt: general – Methods: numerical

## 1. Introduction

When a Transneptunian object (TNO) or a Centaur occults a star, their sizes and shapes can be determined with kilometric accuracy (Elliot et al. 2010; Sicardy et al. 2011; Ortiz et al. 2012; Braga-Ribas et al. 2013) from the resulting light curves of ground-based observers located inside the shadow path or nearly outside it. Such an accuracy in dimensions can only be rivaled by space missions. Also, ring systems (Braga-Ribas et al. 2014a) and atmospheres as tenuous as few nanobars (Widemann et al. 2009; Sicardy et al. 2011; Braga-Ribas et al. 2013) can be detected when present. These parameters are important for the

study of TNOs<sup>1</sup> and, as a consequence, to retrieve the history and evolution of the outer solar system. However, previous to the observation of such an occultation, its prediction (where and when, on Earth, the event can be detected) is essential.

Prediction of stellar occultations requires both accurate position of the occulted star and of the TNO ephemeris. The budget uncertainty of star position and ephemeris must be smaller than the apparent angular size of the body radius, in order for the occultation to be observable at the foreseen location on Earth. The position of the star is initially taken from an astrometric catalogue, taking into account its proper motion, if available. As the proper motion and the astrometry may be inaccurate, the star is

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<sup>1</sup> TNO will indicate both Transneptunian objects and Centaurs hereafter. Centaur objects have their origin often related to the Transneptunian objects.

observed before the occultation in order to refine its position. Depending on the number and quality of the observations, the accuracy of the final position is about 10 mas to 20 mas which is similar to the apparent angular size of the objects.

The ephemeris is obtained after an orbit determination process. It consists in the determination of orbital elements that minimise the O-C *i.e.* the difference between the observed and the computed positions. The ephemeris remains precise during the observational period but starts to diverge after the last observations.

Previous studies about predictions of occultations tried to overcome this ephemeris divergence problem by using more recent observations. Assafin et al. (2012); Camargo et al. (2014) used a constant offset to the ephemeris to refine the predictions. The offset is determined thanks to a set of observations performed from a few months up to a few weeks before the predicted occultation. In practice, the offset is computed as the difference between the observed position and a reference ephemeris at the date of the offset observations. This method assumes that the offset remains constant or varies by less than the body radius until the occultation date, which is only the case when the occultation occurs few days after the offset observations or when the orbit is relatively well determined.

Fraser et al. (2013) highlighted that the offset is not constant and could lead to unreliable predictions. They proposed to refine the orbit using the offset observations and adopted a maximum-likelihood approach to correct the orbital elements. They used only their own observations and not directly the past observations on Minor Planet Center database. As a consequence, their new orbit strongly depends on their new observations. As so, their ephemeris is likely to diverge in a time span shorter than desired.

Finally, the MIT group (Bosh et al., in prep.) also provides predictions of stellar occultations based on a drift and a periodic term for the offset<sup>2</sup>.

In this paper, we present a new numerical procedure called NIMA (Sect. 2) that computes the orbits of TNOs and Centaurs. NIMA consists of a complete process of orbit determination that profits from all available observations of the TNO/Centaur (past observations from MPC, offset observations and unpublished observations, see Sect. 3) and a specific weighting scheme for observations (Sect. 2.3). We present in Sect. 4 the results of the use of NIMA for a set of 51 TNOs and Centaurs selected for their physical, observable and dynamical characteristics. We conclude (Sect. 5) that NIMA is capable of furnishing significantly improved ephemeris for TNOs and Centaurs, allowing for more accurate stellar occultation predictions within a more extended time span, more than any of the other methods used so far.

## 2. NIMA ephemeris

The NIMA<sup>3</sup> ephemeris was originally developed for orbit determination of Near-Earth Asteroids in the context of the detection of Yarkovsky effect (Desmars 2015). It can be used to determine the orbit of any asteroid and, given the similar dynamical conditions, of any TNO or Centaur (see Sect. 2.1).

The NIMA code allows orbit determination and propagation thanks to a numerical integration of the equations of motion. Compared with the version applied to NEAs or with other codes

for orbit determination such as OrbFit<sup>4</sup>, the main difference is on the weighting scheme which is adapted specifically to the prediction of occultations in order to get short term accurate orbit.

The following sections describe the dynamical model, the fitting process and the weighting scheme.

### 2.1. Numerical integration

The dynamical model of a TNO's motion includes the gravitational perturbations of the Sun and the eight planets. All planets are considered as point masses and the Earth-Moon system is considered as a point mass located at the Earth-Moon Barycentre. No other perturbations are required since TNOs are distant objects. For example, by adding the three biggest asteroids (Ceres, Vesta and Pallas) or by adding Pluto in the dynamical model, we noticed only insignificant changes in orbit determination. The masses of the eight planets and their positions are given by JPL ephemeris DE431 (Folkner et al. 2014).

The equations of motion are numerically integrated through a Gauss Radau integrator (Everhart 1985). The equations of variation as described in Lainey et al. (2004) are also integrated in order to determine the partial derivatives of the position and the velocity components related to the components of state-vector ( $c_j$ ) which encompasses the position and the velocity vectors at a specific epoch.

### 2.2. Fitting process

The fitting process consists in the determination of six parameters  $C = (c_j)$  (the state vector) that minimises the residuals  $\Delta Y$  (the difference between observed and computed positions). This determination makes use of a Levenberg-Marquardt algorithm by iterative corrections of each component of the state-vector. For each iteration, the corrections to apply are determined thanks to the partial derivatives (represented by matrix  $A$ ) and the least square method (LSM) (for more information, see for example Desmars et al. 2009).

$$(\widehat{\Delta C}) = (A^T V_{obs}^{-1} A)^{-1} A^T V_{obs}^{-1} \Delta Y \quad (1)$$

In the LSM, a weighting matrix  $V_{obs}$  is required and we specifically discuss of the weighting scheme in the next section.

The normal matrix  $N$  and covariance matrix  $\Lambda_0$  are defined as:  $N = A^T V_{obs}^{-1} A$  and  $\Lambda_0 = N^{-1}$ .

The standard deviation of each parameter ( $c_j$ ) is given by the root square of the diagonal elements of  $\Lambda_0$ . Moreover, the covariance matrix can be linearly propagated at any date  $t$  thanks to the equation:

$$\Lambda(t) = A(t) \Lambda_0 A(t)^T \quad (2)$$

where  $A(t)$  is the matrix of partial derivatives at date  $t$ . Thus this linear relation can provide the estimated precision of the position in the celestial sphere (right ascension and declination) at any date.

<sup>2</sup> <http://occult.mit.edu/research/occultationPredictions.php>

<sup>3</sup> Numerical Integration of the Motion of an Asteroid

<sup>4</sup> The package OrbFit is available on <http://adams.dm.unipi.it/~orbmain/orbfit/>

### 2.3. Weighting scheme

Observed positions have various accuracies and can be correlated. In this context, we have to consider the covariance matrix of the observed positions  $V_{obs}$  that is supposed to be known in the LSM. In practice we neglect correlations, so that the covariance matrix of the observed positions  $V_{obs}$  is considered as a diagonal matrix where the diagonal components are  $\epsilon_i^2 = 1/\sigma_i^2$  and  $\sigma_i^2$  is the estimated variance of the observed position  $i$  (Desmars et al. 2009).

In orbit determination, the main difficulty is to give an appropriate weight to each position. Positions do not contain only a random error but also many systematic errors with several different sources such as the telescope used for observation, the stellar catalogue used for the reduction, etc.

Carpino et al. (2003) and Chesley et al. (2010) discussed weighting schemes and showed the orbit determination improvement by weighting positions according to the observatory and the stellar catalogue used for the reduction.

However, a problem can appear using this weighting scheme when several dozens of observations were performed during a same night in the same observatory. If individual observed positions have a weight  $\epsilon_i$ , the mean weight for the set of positions will be  $\epsilon_i/\sqrt{N}$  where  $N$  is the number of positions per night. The mean weight can become small whereas positions can be biased. In that case, orbit determination will be degraded by such a set of positions. This problem mainly concerns our offset observations (see Sect. 3.2) with an average of 13.1 observations per night whereas in MPC observations (see Sect. 3.1), there are 2.8 observations per night for the studied objects.

To overcome with this problem, we have adopted a specific weighting scheme by taking into account the estimated precision of each position depending of each observatory and stellar catalogue used but also a possible bias due to the observatory.

The estimated variance  $\omega_i$  of each position  $i$  is given by:

$$\omega_i^2 = N_i b_i^2 + \sigma_i^2 \quad (3)$$

where  $N_i$  is the number of observations performed during the same night and in the same observatory than position  $i$ ,  $b_i$  corresponds to the possible bias depending on the stellar catalogue and observatory and  $\sigma_i$  is the estimated precision of individual position  $i$  provided by Chesley et al. (2010) or by Table 2 for some specific cases.

This weighting scheme avoids assigning an artificial strong weight for a night with several dozens of observations. In such a case, the mean variance tends to  $b_i^2$  and not 0. Another interpretation is that we consider that the mean variance for a single night cannot be smaller than  $b_i^2$ .

Estimated bias  $b_i$  and precision  $\sigma_i$  will depend on the type of positions we deal with. For example, positions from MPC will be considered as average positions since the process of reduction for each position is not completely known. On the contrary, our positions performed to determine the offset, will be considered as precise positions since we know exactly how they were reduced and the quality of the stellar catalogue used for reduction.

We have performed many tests to determine appropriate bias and precision for the different types of positions (see Sect. 3). Finally, we empirically adopt the values given in Table 1 for the bias  $b_i$  and the values given in Table 2 for the individual precision of each position  $\sigma_i$ . For positions from MPC, we adopt  $b_i = 300$  mas and  $\sigma_i$  depends on stellar catalogue and observatory and is given by Chesley et al. (2010).

As a comparison, Fraser et al. (2013) used an uncertainty of 40 and 80 mas for their positions whereas the average value for a position from MPC in the AstDys database is about 0.5 arcsec. Consequently, positions from Fraser et al. (2013) have a weight 100 times more important than an average position, which seems not appropriate.

In our study, the maximum precision for a series of several positions in one single night is given by the bias. In this context, our best positions that come from ESO and reduced with WFI catalogue (described below), will have a weight about 50 times better than an average position from MPC. Positions from ESO and reduced with UCAC4 will have a weight 10 times better than average positions.

## 3. Astrometric observations

For TNOs and Centaurs, most of the positions come from Minor Planet Center database. But since about 2007 we also started to observe these objects, in order to check occultation predictions. Obviously, the first computed information to evaluate the orbit status was the average offset between the observed positions and the object ephemeris. For this reason, here and throughout the paper, these kind of observations are called offset observations. Finally, we also used astrometric positions deduced from previous positive occultations.

### 3.1. MPC observations

Minor Planet Center<sup>5</sup> (MPC) is in charge of receiving and distributing the positional measurements of minor planets, comets and outer irregular natural satellites. For one specific object, the MPC gives the file of observations, the orbital elements, the ephemeris, and many other data. Observations are provided by many different observers and observatories (from professional to amateur telescopes) and positions are derived from a variety of reference catalogues and position reduction procedures. Consequently, the quality of the observations is heterogeneous and that is why the use of a weighting scheme in orbit determination, taking into account the quality of positions, is important.

Due to average quality and precision, most of the positions on the MPC database are provided with 5 decimal digits in time that correspond to less than one second of uncertainty which is enough for TNO/Centaurs, with 2 decimal digits in right ascension that corresponds to 150 mas of uncertainty, and with 1 decimal digit in declination that corresponds to 100 mas of uncertainty. Clearly, if this uncertainty is enough for problem of identification and ephemeris of position, it is not for the stellar occultations context that requires a precision of, at least, 50 mas. However, recent observations are sometimes provided with 1 extra decimal digit in time, right ascension and declination.

### 3.2. Offset observations

A useful step in the process of predicting stellar occultations to a large number of TNOs, for a span of some few years, is to properly determine a set of initial predictions. This set of initial predictions must be complete, that is, must contain all possible events of a given TNO involving stars up to a given magnitude, and must be accurate enough to allow for a selection of those ones for which observational efforts to refine the initial prediction are worth employing.

<sup>5</sup> <http://www.minorplanetcenter.net/>

All initial predictions of stellar occultations to the objects presented in Table 4, exception made to (2060) Chiron and (60558) Echeclus, are detailed in Assafin et al. (2012) and (Camargo et al. 2014) and were based on observations made at La Silla (Chile) with the ESO 2.2m telescope equipped with the Wide Field Imager (WFI).

The observational runs had two different purposes: to cover the future sky path of a given TNO and then to observe the TNO itself. The first set of observations aimed at determining candidate stars to be occulted by the TNO. As a by product of the respective observations, catalogues with positions and proper motions were created to serve as an accurate and - most important - dense reference catalogue for astrometry. These were called WFI catalogues. The second set of observations were used to obtain positions of the TNOs and to determine the respective corrections to their ephemerides. The WFI catalogues were used as reference for the astrometry of these images. (2060) Chiron and (60558) Echeclus were included later in our list of objects. Candidate stars to be occulted by them were selected from UCAC4 (Zacharias et al. 2013) and USNO-B1 (Monet et al. 2003) catalogues and consequent observations to refine the position of the candidate stars and to correct their ephemerides were carried out at the Pico dos Dias observatory.

For the initial predictions given by both Assafin et al. (2012) and (Camargo et al. 2014), corrections to ephemerides were done by considering an average offset as determined from the position differences in the sense "observation minus ephemeris". Such correction (offset), as mentioned earlier in the text, was assumed to be constant until a new one was determined with newer observations.

Although good results were obtained with this method, thanks to the efforts of many observers, the use of offsets is not ideal (see, for instance, Fig. 3). This method does not allow for predictions with accuracy better than 30 mas, on the position of the TNO, with an advance greater than six months. Orbit re-determination is a more straightforward solution, and the offset observations are still useful to refine the orbit.

### 3.3. Astrometry from occultations

Once a stellar occultation is detected, besides the physical properties of the object, we can determine its position on the sky relative to the occulted star. From a multi-site observation, the limb fit of the object to the observed occultation light-curves (chords), has as by-product the centre of the body. This position is relative to a given position, usually the occulted star position, so the position of the object's centre at the middle of the event can be directly calculated, with kilometric accuracy.

The absolute position of the object is then dependent (and limited) on the accuracy of the star position, which is usually many times greater than the lib-fit errors. We performed observations of the occulted stars at Pico dos Dias Observatory to reduce this source of uncertainty. The observations were made near the epoch of the occultation, usually used to update the predictions, so proper motion errors were avoided. Astrometric reductions were made using the WFI catalogue when available, otherwise the UCAC4 catalogue was used as reference. Obtained accuracies on the star positions are on the order of 10 mas to 20 mas, which is about the apparent angular size of the TNOs on our list.

Accurate TNO positions from stellar occultation can be obtained even for single-chord detections. In this case, it is not possible to derive the size of the object, but we can use its estimated

size<sup>6</sup> to derive its centre. The observed chord is fitted to a presumed circular object and the centre is calculated with respect to the star position. This will lead to a determination of the centre with an error of few hundreds of kilometres (about the precision of the object size), even considering a north or south solution. The error on the absolute object centre is still dominated by the absolute star position.

This is a very straightforward way to obtain precise TNO positions and was applied to all the detected stellar occultations by TNOs (Braga-Ribas et al. 2014b). This will be specially interesting when GAIA catalogue is available, as the position of the objects will no longer be limited by the accuracy of the star position.

## 4. Results

### 4.1. Comparison between ephemerides

Ephemerides for TNOs can be found on several databases. To make comparisons with the NIMA ephemeris, we take on the main databases: JPL Horizons<sup>7</sup>, VO-Miriade<sup>8</sup>, and AstDys<sup>9</sup> (AstDys makes use of the Orbfite package). Minor Planet Center also provides ephemeris but we did not consider it because the coordinate values are truncated to 0.1s in right ascension and 1 arcsec in declination, which is clearly not enough for predictions of occultations.

Assuming that these databases were produced by using the same set of positions available on the MPC, we can say that they fitted their orbits with the same set of positions as we used with NIMA. We compared the ephemeris for two specific objects: (50000) Quaoar which has a long period of observations (1954-2014) and 2008OG19 with a small observational period (2008-2012). Figures 1 and 2 present the difference between the different ephemerides in right ascension<sup>10</sup>, declination, and geocentric distance during 2010-2020 period. We also use a version of NIMA (nima v0) without changing the weight of positions, i.e. using the weight of Chesley et al. (2010) as in Orbfite. Moreover for Quaoar, we also add the ephemeris from (Fraser et al. 2013) just for information because the ephemeris can not be fully compared with the other ones since it made use of additional observations published in the same paper.

For objects with a large period of observations such as Quaoar, the differences between the ephemerides are small (less than 0.1 arcsec) on the 2010-2020 period. Even with Fraser ephemeris, the difference is small whereas they used additional observations. As they used Orbfite package for orbit determination, the difference without these additional observations, would probably be close to AstDys ephemeris. The small differences between ephemerides for Quaoar indicate the good quality of the orbit, because of the long period of observations available. For objects with a short period of observations such as 2008OG19, the differences between ephemerides are larger (several arcseconds) indicating the low-quality of the orbit.

The difference between ephemerides consists in a secular drift and a 1yr-period term. This periodic term corresponds to the parallax due the Earth's revolution and the difference in distance between the ephemerides.

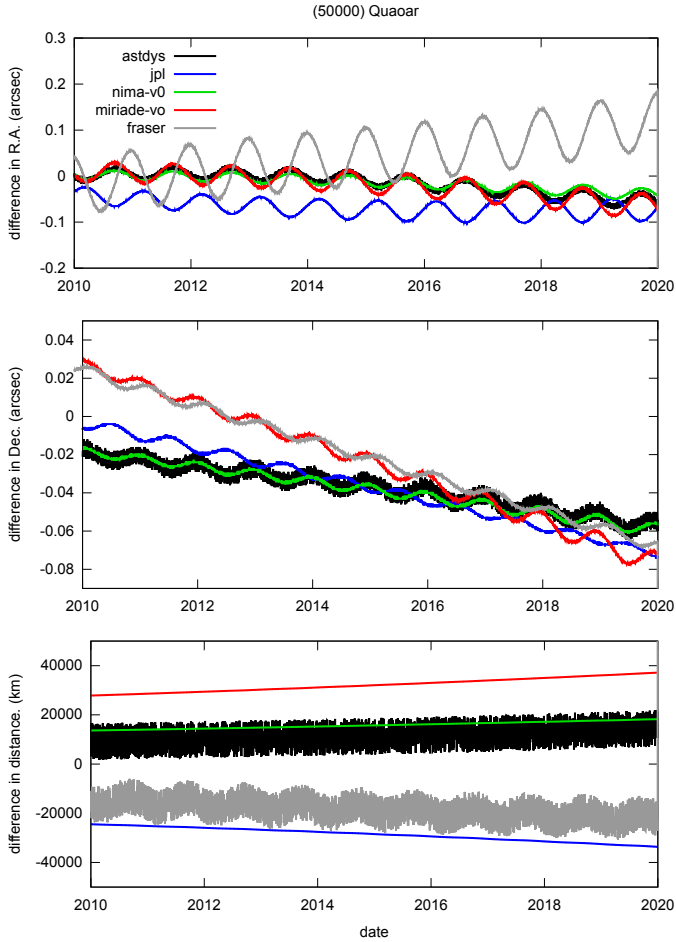
<sup>6</sup> Sizes of objects are from <http://www.johnstonsarchive.net/astro/tnodiam.html>.

<sup>7</sup> <http://ssd.jpl.nasa.gov/horizons.cgi>

<sup>8</sup> <http://vo.imcce.fr/webservices/miriade/>

<sup>9</sup> <http://hamilton.dm.unipi.it/astdys/index.php>

<sup>10</sup> The difference in right ascension is weighted by  $\cos \delta$ .



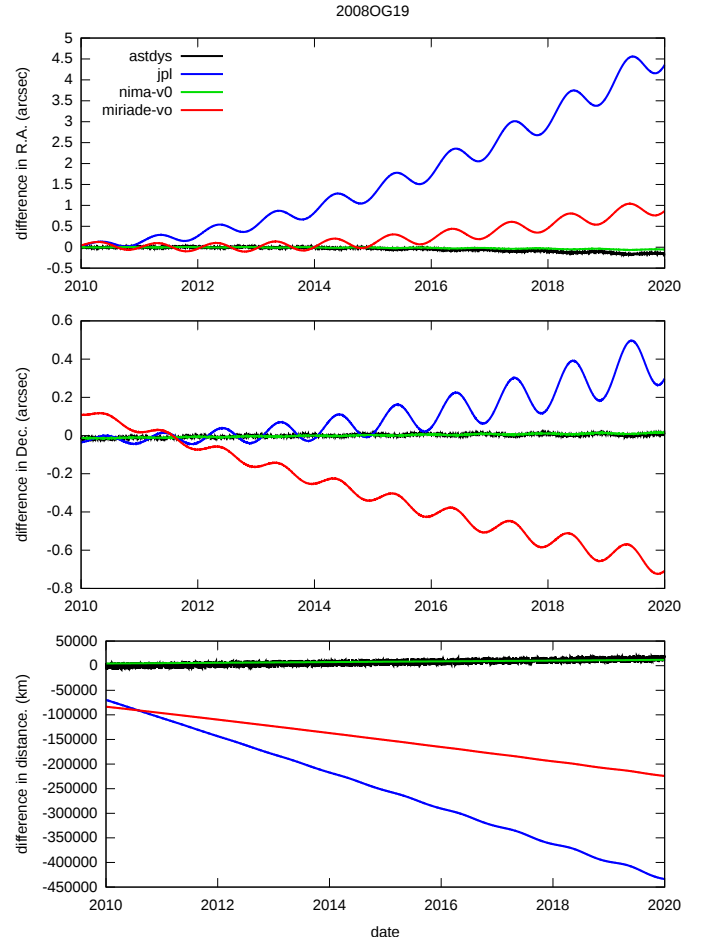
**Fig. 1.** Difference in right ascension weighted by  $\cos \delta$ , declination and geocentric distance between NIMA and each ephemeris for (50000) Quaoar.

By using the classical weighting scheme (with the weights given by Chesley et al. (2010)), we have a very similar orbit between NIMA and OrbFit (corresponding to AstDys ephemeris).

Compared to other ephemerides, the advantages of the NIMA ephemeris are to allow for the use of more observations, not only MPC observations and to have the control of the weighting process.

#### 4.2. Offset variation

By comparing the difference between NIMA ephemeris, that has been fitted to all positions (MPC + Offset observations) and JPL ephemeris determined with only MPC positions, we have an estimation of the offset variation. Figure 3 represents the difference between NIMA and JPL ephemerides for 2004NT33. For this object, we have two additional sets of offset observations made at ESO in November 2012 and May 2013. The sets of observations correspond to one single night of observations where several observations were performed. The blue dots represent the average positions of each set and the error bar represents the standard deviation ( $1-\sigma$ ). Obviously, NIMA ephemeris fits to ESO observations whereas JPL ephemeris does not. As commented in the previous section, the offset variation consists in a secular drift and a 1-year periodic term corresponding to a difference in the heliocentric distance between the two orbits. As expected, the offset is not constant with time. We can notice that the extrema of

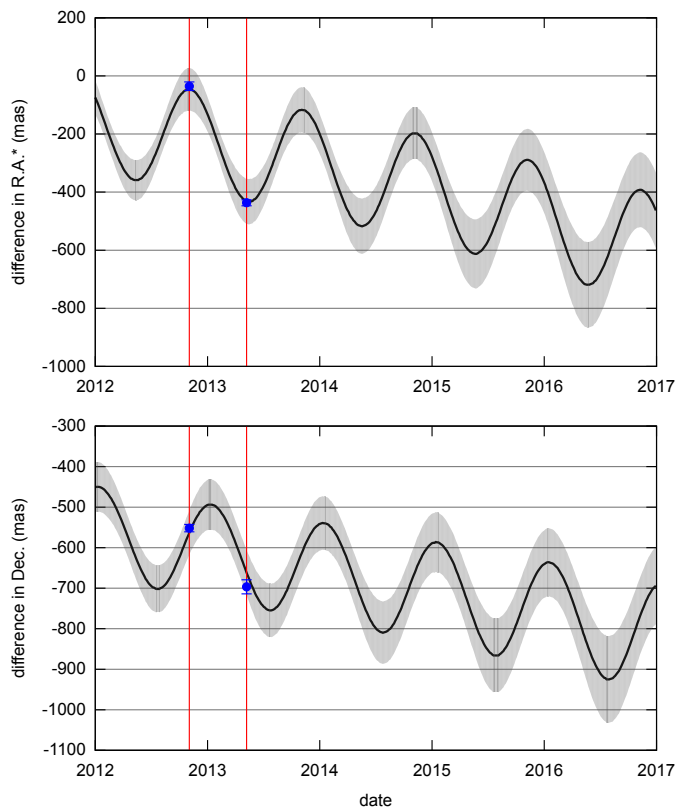


**Fig. 2.** Difference in right ascension weighted by  $\cos \delta$ , declination and geocentric distance between NIMA and each ephemeris for 2008OG19.

the oscillations correspond to the quadrature of the object (when elongation is 90 degrees). The grey area in the figure represents the estimated uncertainty ( $1-\sigma$ ) of the NIMA ephemeris and we can notice that the uncertainty increases with the time.

#### 4.3. Predictions of occultations

By design, the NIMA ephemeris is most suited for occultation predictions, as its better accuracy allows for more confident predictions farther in advance, for occultations to occur several months after the offset observations. We compare two predictions for the occultation by (28978) Ixion on 24 June 2014 with the offset method and with the NIMA ephemeris. Figure 4 shows the different predictions. The path of the shadow crosses over the North of Australia for the offset method and the Centre of Australia for the NIMA ephemeris. Actually, the occultation was successfully observed in the Centre of Australia (indicated by the green point), indicating that the prediction with NIMA ephemeris was more accurate. In fact, since July 2013 when NIMA ephemeris was applied to the occultation predictions until February 2015, 10 TNOs and 3 Centaurs events have been detected whereas 13 occultations by TNOs have been detected between 2009 and 2012.

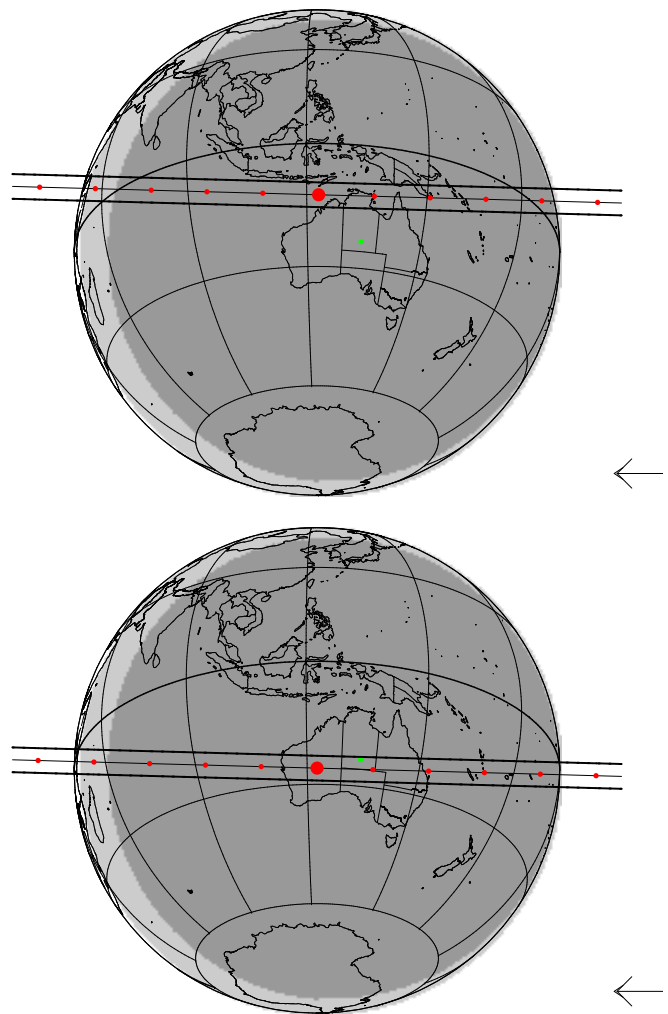


**Fig. 3.** Difference between NIMA and JPL ephemerides for 2004NT33 in right ascension (weighted by  $\cos \delta$ ) and declination during 2012–2017. The grey area represents the uncertainty of NIMA ephemeris and the blue bullets and their error bars represent the offset observations used for NIMA ephemeris.

#### 4.4. Precise positions from occultations

As explained in Sect. 3.3, precise astrometric positions of TNOs can be deduced from previous positive occultations. We have reduced 14 astrometric positions for 8 different objects so far. Table 3 presents the residuals in right ascension weighted by  $\cos \delta$  ( $RA^*$ ) and in declination of these positions. Most of the positions have a precision less than 50 mas.

These positions can be used to refine the orbit and to improve the prediction of stellar occultations. For example, three positive occultations by (50000) Quaoar have been observed in 2011 and 2012 (Braga-Ribas et al. 2013). To show that the positions provided by these occultations help to improve the orbit quality, we present the prediction of the third observed event, on 15 October 2012, with two different sets of positions. The first, using all the available observations until this date (including offset observations); and the second using the same observations plus the two previous positions deduced from the previous detected occultations, on May 2011 and February 2012. We created the prediction maps for the two cases, see Figure 6. Figure 5 shows the difference between NIMA and JPL ephemerides in right ascension (weighted by  $\cos \delta$ ) and declination for these two cases. The difference between the two orbits is about 50 mas in declination at the date of the occultation. For the configuration of this event, an uncertainty in right ascension corresponds to mainly to an uncertainty in time of occultation, whereas an uncertainty in declination corresponds to an uncertainty in the location of the path, which is more important for observation purposes. The two orbits correspond to two different paths of the shadow for



**Fig. 4.** Prediction map of the occultation by Ixion on 24 June 2014 with the offset method (top) and with the NIMA ephemeris (bottom). The occultation was detected in the place indicated by the green point.

the occultation on 15 October 2012 (Fig. 6). The first one using only the observations predicts an occultation by Quaoar in the South of Peru whereas the second orbit using observations and positions of occultations predicts an occultation over Chile. Finally, the occultation was positively detected at Cerro Tololo by the PROMPT telescopes, in the Centre of Chile (green point on the figure), showing that the prediction was better thanks to the position given by previous occultations.

#### 4.5. Discussion

As the position deduced from occultations is only affected by the error on the position of the occulted star, the derived position is more accurate than classical observations. In particular, these positions can highlight systematic errors in observations. Figure 5 reveals systematic errors in the positions from the two last sets of observations performed on May and July 2012. Even if the positions of previous occultations are taken into account, the orbit cannot match accurately these observed positions. The reason comes from the quality of all the positions used to make the orbit. The change of 80 mas in right ascension and 90 mas in declination between May and July 2012 may only be explained

**Table 3.** Residuals of 14 astrometric positions deduced from previous occultations. The table indicates the name of the object, the date of the occultation, the weight used in orbit determination in mas, and the residuals in arcsec in RA\* and Dec. The weight depends mostly on the quality of star position determination.

Name	Date	Weight	Residuals	
			RA*	Dec
2002KX14	2012-04-26	90	-0.057	-0.003
2003AZ84	2011-01-08	40	-0.017	-0.009
	2012-02-03	40	-0.006	0.011
2003VS2	2014-03-04	75	0.004	-0.015
Chariklo	2013-06-03	40	-0.006	-0.020
	2014-02-16	40	0.006	0.039
	2014-04-29	40	0.029	-0.011
	2014-06-28	40	-0.030	0.009
Eris	2013-08-29	75	0.006	0.007
Makemake	2011-04-23	75	-0.010	0.083
Quaoar	2011-05-04	40	0.005	-0.008
	2012-02-17	40	0.000	-0.033
	2012-10-15	40	-0.003	-0.002
Varuna	2013-01-08	40	0.014	-0.005

by systematic errors. They may come from zonal errors in stellar catalogue, the telescope, the sky conditions, etc.

As a comparison, the positions from the offset observations on Fig. 3 may also be affected by systematic errors but since there is a few number of observations for 2004NT33, the two additional positions fit well the NIMA orbit (black line). In that case, systematic errors are hard to detect and we can only trust the observations. The main difference of this study is that the orbit determination as well as the uncertainty of the ephemeris now take into account possible systematic errors in the positions through the adopted weighting scheme.

Systematic errors in the positions are currently the main limiting factor for accurate orbit determination. Some systematic errors are linked to observation such as the zonal error in the catalogue or the differential chromatic refraction, and they may be partially corrected thanks to the Gaia catalogue. Other are linked to dynamics such as the difference between the positions of the photocentre and the barycentre for binary systems, and may be only corrected with a careful modelling of the mutual orbit of the two bodies.

The theory of orbit determination allows to deal with positions with different precisions by using a weighting scheme but not with systematic errors. Until systematic errors in positions from CCD images can be brought to a minimum, thanks to the astrometry from Gaia, the weighting scheme used in this study allows to partially deal with these errors.

#### 4.6. TNO's ephemerides and observations

As output of this study, we make available the ephemeris of the 51 selected TNOs and Centaurs during the period 2010-2020. Used to make predictions of stellar occultations, these ephemerides are available in bsp file usable with the SPICE library (Acton 1996) at the address <http://www.imcce.fr/~desmars/research/tno/>. These ephemerides will be regularly updated once new observations become available.

The predictions of forthcoming stellar occultations are available at <http://devel2.linea.gov.br/~braga.ribas/campaigns/> or <http://www.lesia.obspm.fr/perso/>

[bruno-sicardy/](http://bruno-sicardy/). For comparisons, predictions for past occultations are available at <http://devel2.linea.gov.br/~braga.ribas/campaigns/old.html>.

The offset observations of the selected TNOs are available on the CDS. The statistics of the residuals, the number and the time-span of MPC observations and offset observations are given in Table 4. The offset observations have a better quality than MPC positions and generally they help to extend the period of positions.

## 5. Conclusion

The prediction of stellar occultations by TNOs is, and will be for a long time, thoroughly dependent on observations. Although the astrometry from the GAIA space mission will provide star positions with an accuracy from few microarcseconds to hundreds of microarcseconds, observations aiming at the determination of positions of TNOs will still be necessary for precise orbit calculation purposes. In this context, a large contribution is expected from deep-sky surveys, such as Pan-STARRS<sup>11</sup> or the LSST<sup>12</sup>. This survey will repeatedly observe the sky southern up to  $\delta = +10$  degrees in 6 bands, also providing multiple observations for tens of thousands of TNOs. NIMA is a suitable tool to ingest these data and provide improved ephemerides for these objects.

On the other hand, the astrometry and photometry from GAIA will greatly improve the astrometric reduction of CCD and photographic images as well as renew the importance of old epoch images. In fact, old plates with solar system objects may be reduced with reference star positions having sub-mas accuracies at the plates' epoch. These old epoch positions are of utmost importance to the accuracy of ephemerides for objects with long (hundreds of years) periods. In this context, it should be mentioned that our team has an image database with solar system objects, acquired at the Pico dos Dias Observatory, that spans about 20 years. Those with TNOs, from Pico dos Dias and La Silla, span half of this time. All of them will be re-reduced with the GAIA astrometric catalogue.

Another source of ephemeris improvement could also be obtained from the observational strategy. Errors in the TNO's distance may amount to thousands of kilometres and reflect as one year period oscillations in plots showing position differences between different ephemerides of the same object. Here, it should be noted that observations are preferably made close to opposition, a configuration that is less sensitive to parallax effects in position when compared to quadrature. Therefore, more frequent observations of TNOs in quadrature would improve the accuracy of their ephemerides.

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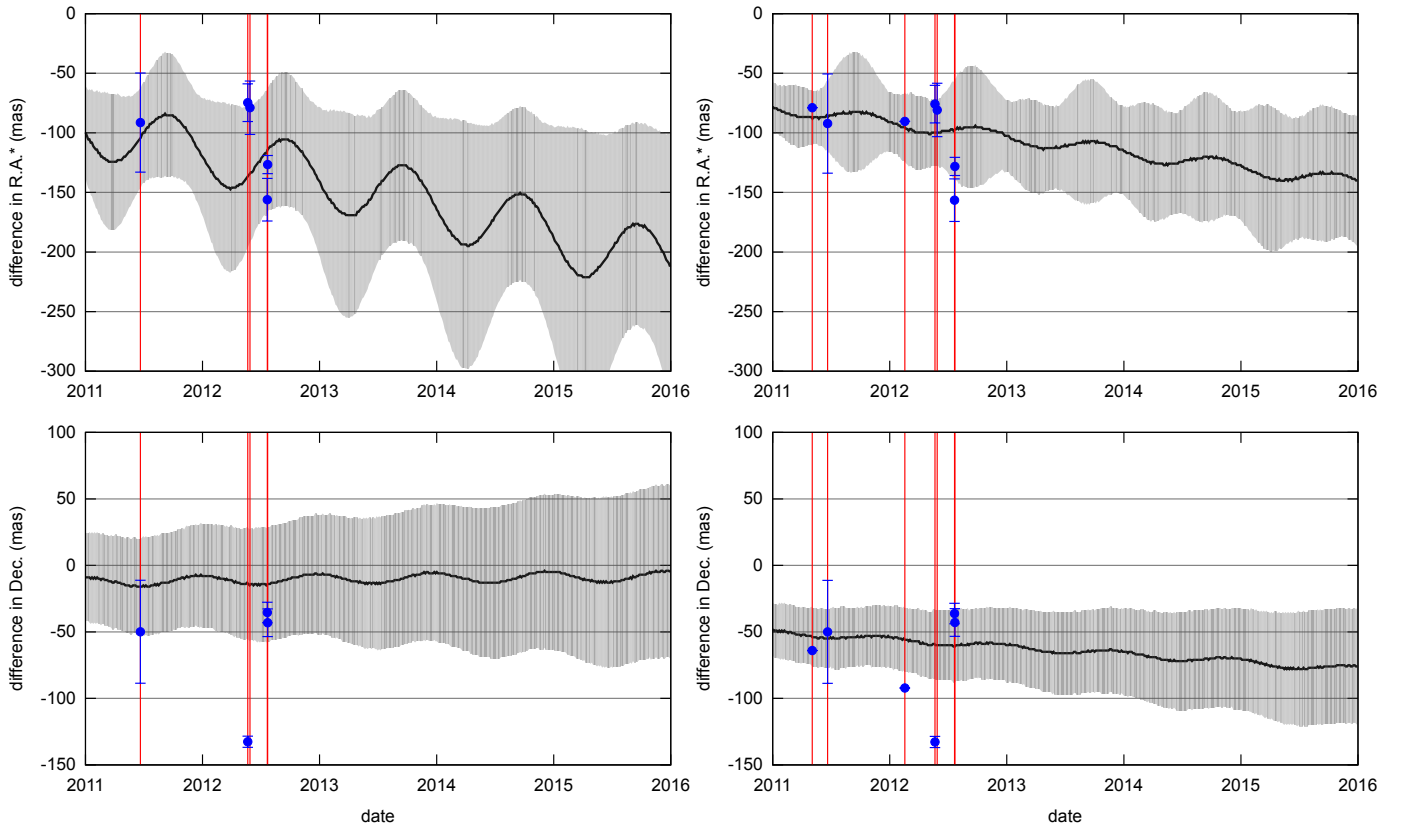


**Table 1.** Estimated bias for offset observations in both right ascension and declination for different stellar catalogues and observatories

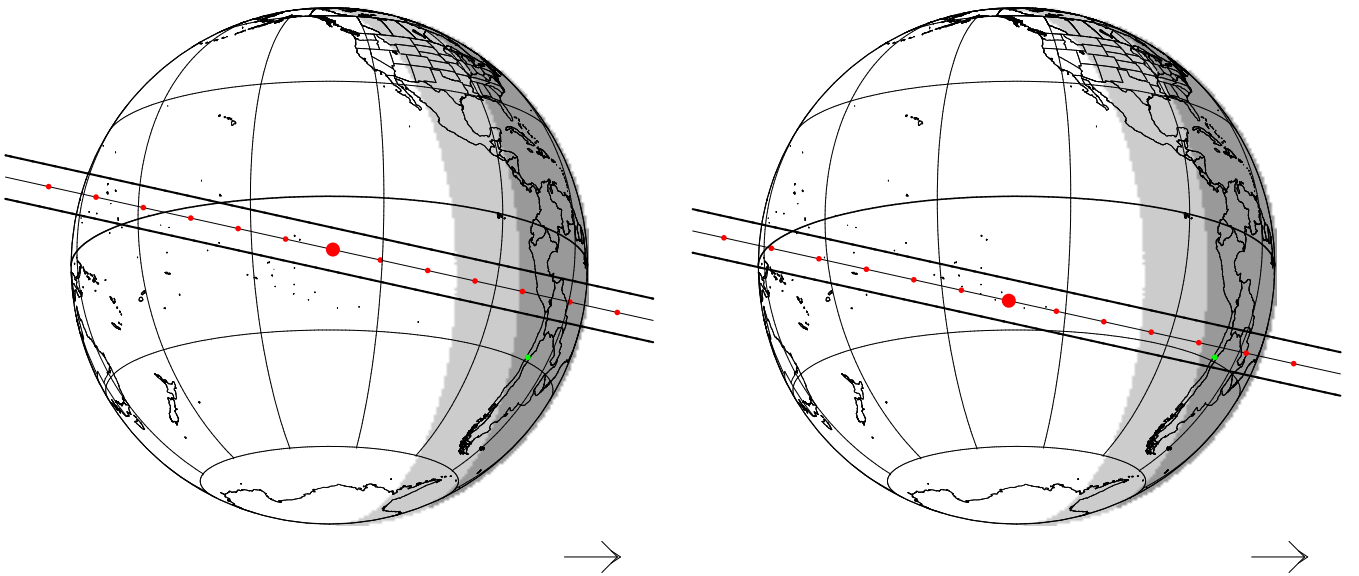
Source	Catalogue	IAU code	Observatory	estimated bias $b_i$ (in mas)
MPC	—	—	all	300
Offset	WFI	809	European South Observatory	75
		874	Observatório do Pico dos Dias	150
		586	Pic du Midi	150
		493	Calar Alto	150
		J86	Sierra Nevada	150
		I95	La Hita	150
		Z20	Mercator La Palma	150
		J13	Liverpool La Palma	150
		—	Other	300
	UCAC/other	809	European South Observatory	150
		874	Observatório do Pico dos Dias	225
		586	Pic du Midi	225
		493	Calar Alto	225
		J86	Sierra Nevada	225
		I95	La Hita	225
		Z20	Mercator La Palma	225
Occultation	—	244	Geocenter	0
		—	Other	300
Fraser et al. (2013)	2MASS/SDSS	267	CFHT	300
		568	Gemini-Mauna Kea	300

**Table 2.** Estimated precision for offset observations in both right ascension and declination for different observatories

Source	IAU code	Observatory	estimated precision $\sigma_i$ (in mas)
MPC	—	all	Chesley et al. (2010)
Offset	809	European South Observatory	bias
	874	Observatório do Pico dos Dias	bias
	586	Pic du Midi	bias
	493	Calar Alto	bias
	J86	Sierra Nevada	bias
	I95	La Hita	bias
	Z20	Mercator La Palma	bias
	J13	Liverpool La Palma	bias
	—	Other	bias
Occultation	244	Geocenter -accurate position-	40
		Geocenter -approximate position-	75
Fraser et al. (2013)	267	CFHT	bias
	568	Gemini-Mauna Kea	bias



**Fig. 5.** Difference between NIMA and JPL ephemerides for (50000) Quaoar in right ascension weighted by  $\cos \delta$  (top) and declination (bottom) during 2011-2016, by using only the positions until October 2012 (left) and by using the positions until October 2012 and plus the two positions from the two previous occultations in May 2011 and February 2012 (right). The grey area represents the uncertainty of NIMA ephemeris and the blue bullets and their error bars represent the positions from the offset observations used for NIMA ephemeris.



**Fig. 6.** Prediction map of the occultation by Quaoar on 15 October 2012 using only observations (left) and using observations and 2 previous occultations (right).

**Table 4.** Statistics of post-fit residuals for the 51 selected TNO and Centaur for MPC positions (first line) and for the positions from the offset observations (second line) used for orbit determination. The mean  $\mu$  and the standard deviation  $\sigma$  of right ascension weighted by  $\cos \delta$  and of the declination are indicated as well as the number of accepted positions and the time span.

TNO	$\mu_{\alpha^*}$	$\sigma_{\alpha^*}$	$\mu_{\delta}$	$\sigma_{\delta}$	number	time-span
(24835) 1995SM55	-0.047	0.683	0.017	0.459	125	1982-2012
	0.006	0.019	-0.001	0.043	10	2012
(26375) 1999DE9	0.006	0.468	-0.052	0.294	71	1990-2008
	-0.022	0.056	0.009	0.033	40	2012-2013
(47171) 1999TC36	-0.034	0.519	-0.046	0.545	106	1974-2013
	0.010	0.030	0.014	0.019	37	2012-2013
(55565) 2002AW197	0.081	0.214	0.034	0.190	115	1997-2013
	-0.006	0.047	0.013	0.054	55	2012-2013
(119951) 2002KX14	0.081	0.196	0.030	0.218	56	1984-2011
	0.039	0.035	-0.017	0.030	36	2012-2013
(307261) 2002MS4	0.046	0.297	0.028	0.382	58	1954-2009
	0.002	0.046	0.011	0.031	47	2012-2014
(84522) 2002TC302	0.013	0.491	0.012	0.327	96	2000-2013
	-0.014	0.071	0.004	0.070	125	2011-2014
(55636) 2002TX300	-0.003	0.234	0.016	0.306	341	1954-2013
	-0.070	0.028	0.002	0.011	10	2013
(55637) 2002UX25	-0.058	0.342	-0.047	0.429	74	1991-2013
	-0.009	0.057	0.066	0.065	31	2012
(55638) 2002VE95	-0.008	0.254	0.002	0.278	193	1990-2013
	-0.018	0.044	-0.010	0.019	23	2012-2013
(119979) 2002WC19	0.036	0.257	-0.016	0.364	74	2001-2012
	0.018	0.061	-0.036	0.057	24	2012-2013
(208996) 2003AZ84	0.037	0.384	0.018	0.351	103	1996-2014
	0.012	0.062	0.009	0.067	79	2011-2013
(120132) 2003FY128	0.093	0.307	0.053	0.410	68	1989-2012
	0.001	0.035	-0.008	0.028	32	2012-2013
(120178) 2003OP32	0.004	0.437	0.034	0.356	68	1990-2011
	-0.003	0.028	-0.002	0.021	59	2012-2013
2003UZ41	0.111	0.322	0.123	0.418	36	1954-2010
	-0.008	0.017	-0.006	0.013	10	2012
(84922) 2003VS2	-0.024	0.294	0.038	0.364	177	1991-2014
	0.013	0.077	-0.020	0.050	37	2011-2014
(90568) 2004GV9	0.034	0.493	0.023	0.472	62	1954-2011
	0.001	0.012	0.006	0.013	18	2013
2004NT33	0.003	0.346	0.182	0.508	27	1982-2010
	-0.020	0.069	0.005	0.068	25	2011-2013
(175113) 2004PF115	0.067	0.326	0.099	0.323	37	1992-2010
	-0.005	0.012	-0.007	0.008	10	2012
(120348) 2004TY364	-0.029	0.312	0.091	0.404	20	1983-2005
	-0.001	0.015	-0.001	0.017	14	2012-2013
(144897) 2004UX10	0.059	0.352	0.149	0.572	83	1953-2007
	0.002	0.016	-0.028	0.020	8	2012
2011FX62-2005CC79	0.100	0.374	0.082	0.331	34	2002-2012
	-0.002	0.032	-0.012	0.025	29	2012-2013
(303775) 2005QU182	-0.082	0.217	0.008	0.304	81	1974-2011
	0.025	0.037	-0.027	0.053	9	2012
(145451) 2005RM43	0.004	0.200	-0.028	0.198	206	1976-2014
	0.008	0.022	-0.005	0.019	10	2012
(145452) 2005RN43	-0.014	0.171	-0.011	0.148	314	1954-2013
	0.019	0.012	0.015	0.008	10	2012
(145453) 2005RR43	0.009	0.193	-0.029	0.201	221	1976-2014
	-0.005	0.011	0.004	0.014	10	2012
(202421) 2005UQ513	-0.130	0.439	-0.021	0.355	63	1990-2013
	0.047	0.098	0.003	0.048	55	2012-2014
2007JH43	0.052	0.249	0.036	0.348	45	1984-2012
	-0.018	0.055	-0.009	0.026	47	2012-2013

**Table 4.** continued.

TNO	$\mu_{\alpha^*}$	$\sigma_{\alpha^*}$	$\mu_{\delta}$	$\sigma_{\delta}$	number	time-span
(278361) 2007JJ43	0.041	0.280	0.159	0.194	104	2002-2012
	-0.005	0.015	-0.024	0.014	22	2013
(225088) 2007OR10	0.023	0.281	0.039	0.351	71	1985-2011
	-0.006	0.045	0.001	0.027	12	2012
(229762) 2007UK126	-0.016	0.215	0.053	0.272	73	1982-2013
	-0.001	0.012	-0.026	0.010	10	2012
2008OG19	-0.012	0.312	-0.050	0.495	27	2008-2012
	0.000	0.016	0.002	0.015	18	2012-2013
2010EK13	0.109	0.307	0.078	0.158	123	2002-2011
	-0.025	0.028	-0.042	0.055	30	2012-2013
(55576) Amycus	0.007	0.697	-0.027	0.514	76	1987-2007
	-0.023	0.058	-0.006	0.020	28	2013
(8405) Asbolus	-0.010	0.533	-0.018	0.510	466	1995-2011
	-0.012	0.129	-0.011	0.109	13	2012-2013
(54598) Bienor	-0.024	0.413	-0.009	0.343	167	1975-2013
	0.011	0.037	0.017	0.052	69	2012-2014
(10199) Chariklo	0.058	0.562	0.027	0.476	571	1988-2011
	-0.010	0.074	0.009	0.039	336	2011-2014
(2060) Chiron	-0.011	0.576	-0.033	0.593	1304	1895-2014
	0.088	0.152	-0.062	0.078	78	2014
(83982) Crantor	-0.027	0.380	0.011	0.423	116	2001-2014
	-0.001	0.008	-0.024	0.008	8	2013
(60558) Echeclus	0.053	0.553	0.060	0.490	686	1979-2014
	–	–	–	–	0	–
(136199) Eris	0.016	0.317	0.015	0.256	615	1954-2014
	-0.059	0.068	-0.004	0.074	96	2007-2012
(136108) Haumea	0.022	0.313	0.014	0.284	1139	1955-2014
	0.021	0.051	-0.001	0.104	148	2011-2013
(38628) Huya	0.013	0.500	0.044	0.497	151	1996-2014
	-0.008	0.018	-0.001	0.018	22	2013
(28978) Ixion	-0.022	0.319	-0.025	0.340	172	1982-2014
	0.026	0.045	-0.013	0.060	216	2009-2014
(136472) Makemake	-0.013	0.418	0.023	0.281	1081	1955-2014
	0.000	0.067	-0.058	0.070	484	2009-2013
(90482) Orcus	0.022	0.275	-0.012	0.201	434	1951-2014
	-0.034	0.045	0.006	0.047	58	2011-2014
(50000) Quaoar	0.004	0.387	0.025	0.344	400	1954-2014
	-0.003	0.043	-0.017	0.047	111	2011-2013
(120347) Salacia	-0.004	0.227	0.034	0.210	69	1982-2010
	0.008	0.013	-0.013	0.016	12	2012
(90377) Sedna	-0.084	0.536	0.101	0.523	91	1990-2012
	0.086	0.089	-0.044	0.044	23	2012
(174567) Varda	-0.006	0.332	0.049	0.354	72	1980-2010
	-0.006	0.032	-0.002	0.021	29	2013-2014
(20000) Varuna	-0.077	0.422	0.079	0.372	301	1954-2014
	-0.007	0.051	-0.009	0.028	172	2012-2013